

# Local Context in Nonlinear Deformation Models for Handwritten Character Recognition

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# Overview

**Introduction**

**State-of-the-Art**

**Nonlinear Matching**

**Local Context**

**Experiments and Results**

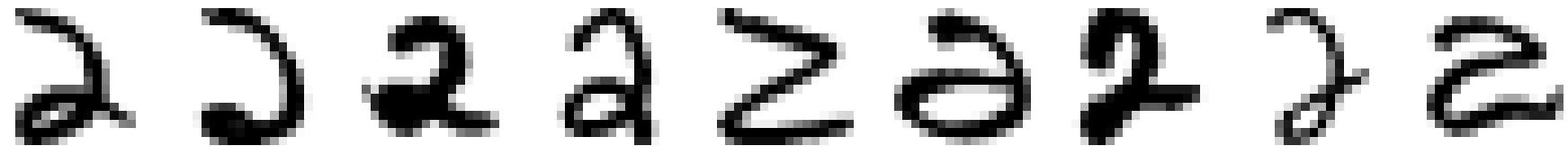
**Conclusion**

# Introduction

**object recognition**  $\Rightarrow$  suitable models of image transformation important

**recognition of handwritten characters is prototypical:**

- different writing styles  $\Rightarrow$  varying appearances
- reference model for each class is well-defined



evaluate two-dimensional **nonlinear deformation** models  
(true two-dimensional, pseudo-two-dimensional, zero-order)  
using **local image context** of each pixel

hope: fewer 2D constraints can be compensated by using local context  
 $\Rightarrow$  confirmed by experimental results

very competitive results across five different tasks (e.g. 0.54% on MNIST)

# State-of-the-Art in Character Recognition

**Belongie & Malik<sup>+</sup> 2002 (Berkeley) — shape context matching:**  
**shape contexts = log-polar histograms of contour points**  
**iterative matching with 2D-splines and the Hungarian algorithm**

**DeCoste & Schölkopf<sup>+</sup> 2002 (Caltech / MPI Tübingen) — invariant SVM:**  
**use virtual data and kernel jittering in support vector machine**

**Simard 2003 (Microsoft Research) — convolutional neural net:**  
**generate large amount of virtual data on the fly (e.g. factor 1000)**  
**during training of a well-designed neural network**

# Nonlinear Matching

**test image**  $A = \{a_{ij}\}$       **reference image**  $B = \{b_{xy}\}$        $a_{ij}, b_{xy} \in \mathbb{R}^U$

**image deformation mapping**  $(x_{11}^{IJ}, y_{11}^{IJ}) : (i, j) \mapsto (x_{ij}, y_{ij})$

**mappings must fulfill constraints:**  $(x_{11}^{IJ}, y_{11}^{IJ}) \in \mathcal{M}$

**decision rule:**

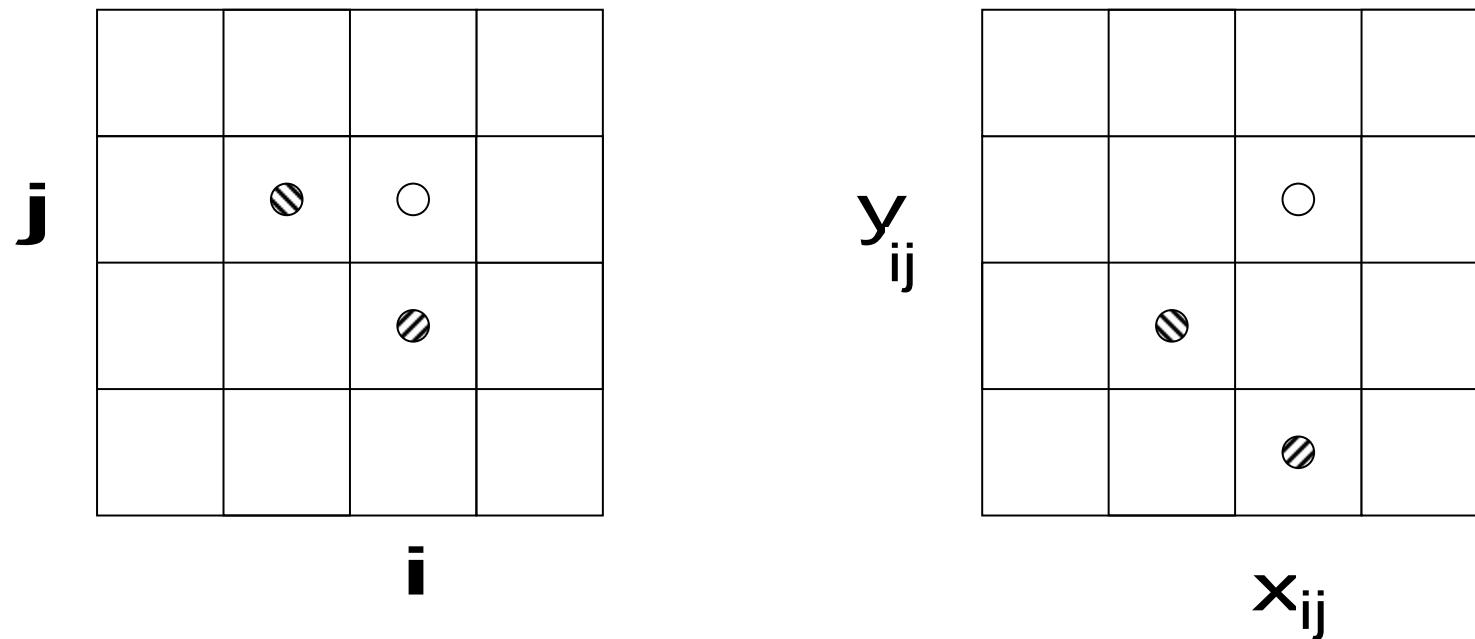
$$r(A) = \arg \min_k \left\{ \min_{n=1, \dots, N_k} d(A, B_{nk}) \right\}$$

$$d(A, B) = \min_{(x_{11}^{IJ}, y_{11}^{IJ}) \in \mathcal{M}} \left\{ d'(A, B_{(x_{11}^{IJ}, y_{11}^{IJ})}) \right\}$$

$$d'(A, B_{(x_{11}^{IJ}, y_{11}^{IJ})}) = \sum_{i,j} \sum_u ||a_{ij}^u - b_{x_{ij} y_{ij}}^u||^2$$

# 2D Dependencies

**image deformation mapping**  $(x_{11}^{IJ}, y_{11}^{IJ}) \in \mathcal{M} : (i, j) \mapsto (x_{ij}, y_{ij})$



# Models

informal descriptions of the used models:

- |         |   |
|---------|---|
| 2DW     | <b>2-Dimensional Warping (order 2)</b><br>complete 2D constraints, minimization NP-complete   |
| P2DHMM  | <b>Pseudo 2-Dimensional Hidden Markov Model (order 1)</b><br>match columns on columns, regard columns as independent                |
| P2DHMDM | <b>Pseudo 2-Dimensional Hidden Markov Distortion Model (order 1)</b><br>allow additional horizontal displacements in P2DHMM         |
| IDM     | <b>Image Distortion Model (order 0)</b><br>disregard relative displacements of neighboring pixels<br>restrict absolute displacement |

# Examples

'different classes'

2DW



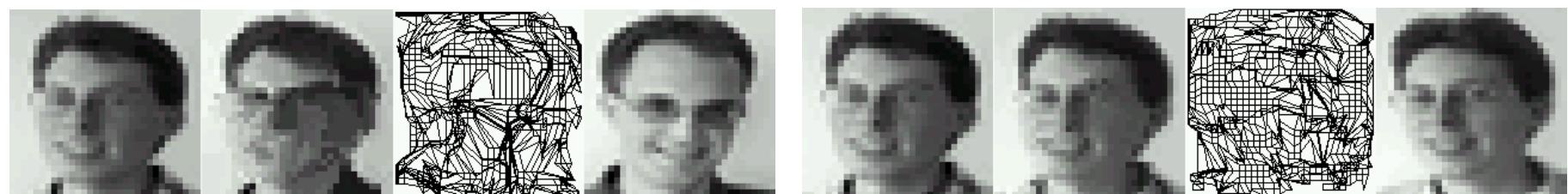
P2DHMM



P2DHMDM



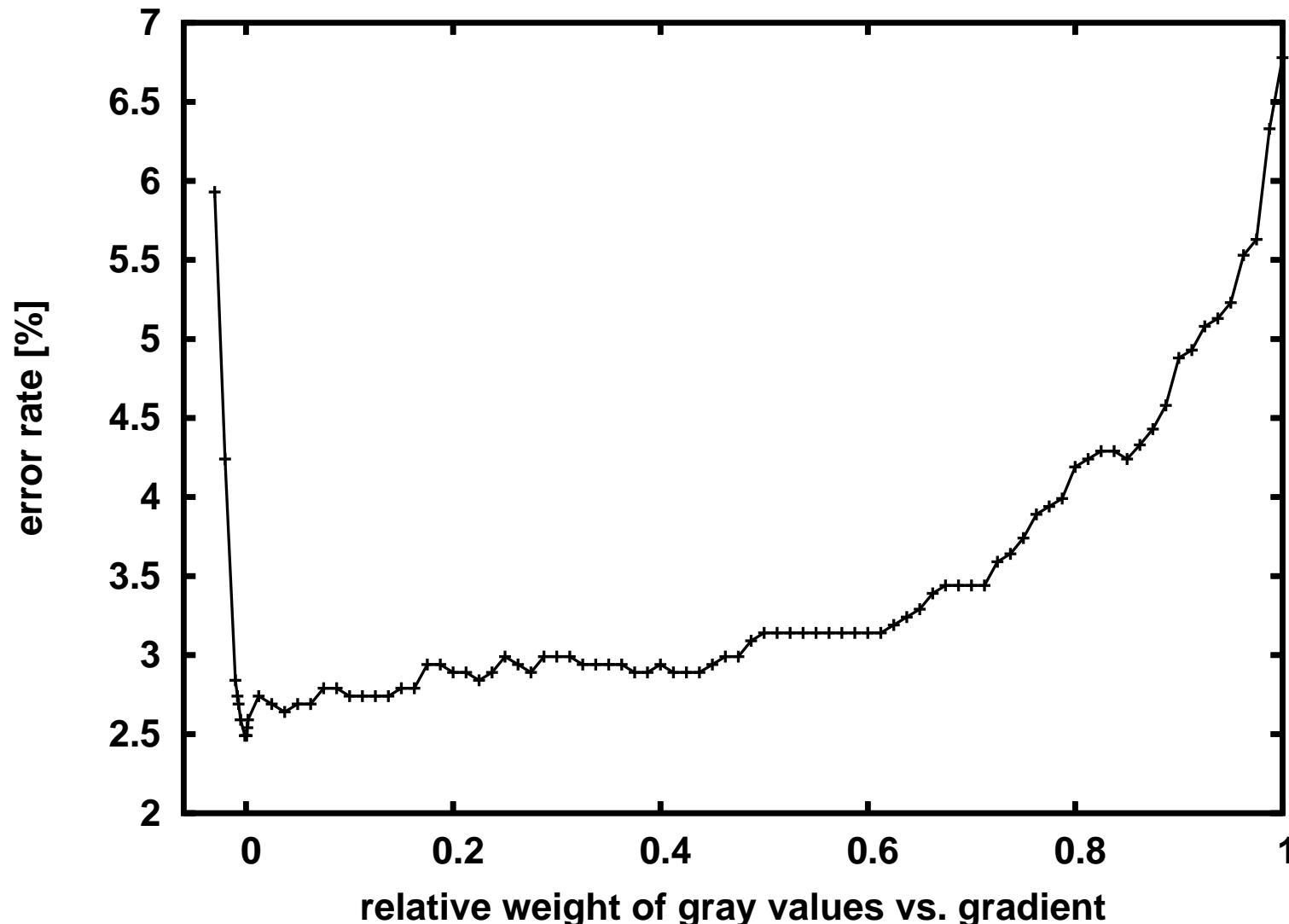
IDM



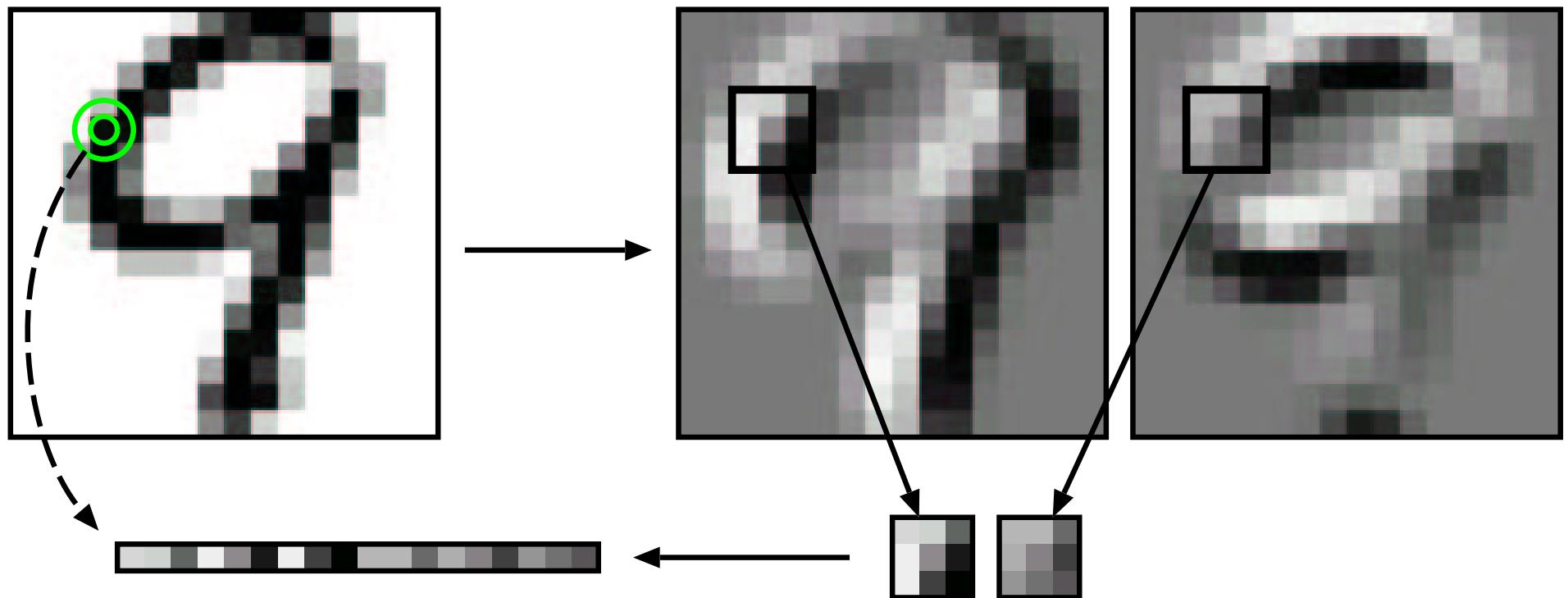
'same class'

# Local Context 1: Image Gradient

error rate on USPS for P2DHMM using gray values and image gradients (Sobel)



# Local Context 2: Windows



# Experiments: Character Corpora

name	example images	size	# train	# test
USPS		$16 \times 16$	7 291	2 007
UCI		$8 \times 8$	3 823	1 797
MCEDAR		$8 \times 8$	11 000	2 711
MNIST		$28 \times 28$	60 000	10 000
ETL6A		$64 \times 63$	15 600	13 000

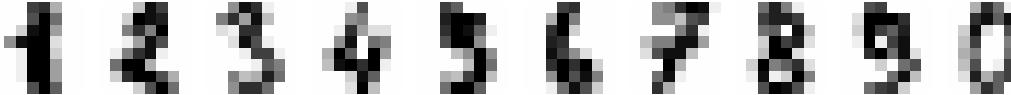
all matching experiments use  $3 \times 3$  context of gradients

# USPS

name	example images	size	# train	# test
USPS		$16 \times 16$	7 291	2 007

method	ER[%]
no matching, 1-NN	5.6
...	...
invariant SVM	external 3.0
2DW, 1-NN	RWTH 2.7
tangent distance, KD, virtual data	RWTH 2.4
IDM, 1-NN	RWTH 2.4
extended SVM training	external 2.2
local features + tangent distance	RWTH 2.0
P2DHMDM, 3-NN	RWTH 1.9

## UCI

name	example images	size	# train	# test
UCI		8×8	3 823	1 797

method	ER[%]
no matching, 1-NN	2.0
PCA mixture external	1.5
P2DHMDM, 1-NN RWTH	0.8
IDM, 1-NN RWTH	0.8

# MCEDAR

name	example images	size	# train	# test
MCEDAR		$8 \times 8$	11 000	2 711

method	ER[%]
no matching, 1-NN	5.7
PCA	4.9
factor analysis	4.7
probabilistic PCA	4.6
IDM, 3-NN	RWTH 3.5
P2DHMDM, 3-NN	RWTH 3.3

# MNIST

name	example images	size	# train	# test
MNIST		$28 \times 28$	60 000	10 000

method	ER[%]
no matching, 1-NN	3.1
deslant, Euclidean distance, $k$ -NN external	2.4
tangent distance, KD, virtual data RWTH	1.0
distortions, neural net, boosting external	0.7
shape context matching, 3-NN external	0.63
invariant SVM external	0.56
IDM, 3-NN*	RWTH 0.54
distortions+, neural net*	external 0.42
combination of best four systems	0.35

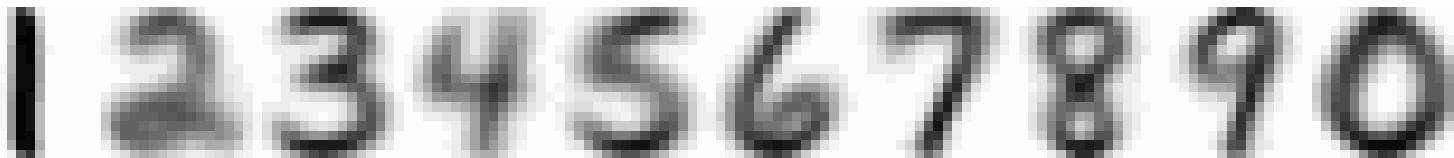
\*: differences not statistically significant according to bootstrap analysis

# ETL6A

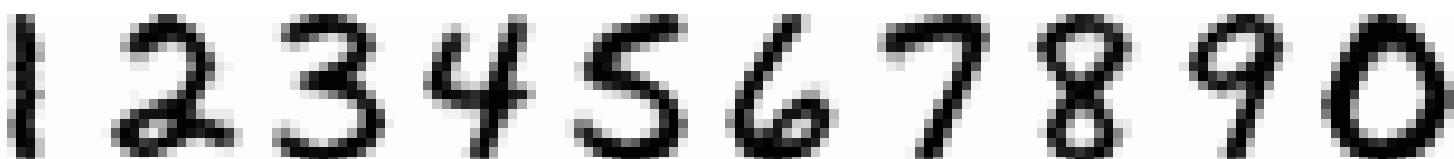
name	example images	size	# train	# test
ETL6A	A B C D E F ... X Y Z	64×63	15 600	13 000

method	ER[%]
no matching, 1-NN	4.5
piece-wise linear 2D-HMM external	0.9
Eigen-deformations external	0.5
IDM, 3-NN RWTH	0.5

# Learning Prototypes



means without deformation



means with deformation

USPS error rates [%] in key experiment

distance	deformation in mean calculation	
	no	yes
no deformation	18.6	26.1
P2DHMDM	25.3	4.9

best other single-prototype result: 5.7%  
(Gaussian single densities, discriminative training, no deformation)

# Conclusions

## Conclusions

- performance increase with local context for all models
- context information  $\Rightarrow$  very good results even with simple distortion models
- excellent results for handwritten character recognition, generalizes across tasks
- [combination of four best systems: 0.35% on MNIST]

software + parameter examples can be downloaded from:  
<http://www-i6.informatik.rwth-aachen.de/~keysers>

# Thank you for your attention.

**<http://www-i6.informatik.rwth-aachen.de/~keysers>**

# Matching Constraints

model (order)	restrictions on $(x_{11}^{IJ}, y_{11}^{IJ})$
2DW (2)	$x_{1j} = 1, x_{Ij} = X, y_{i1} = 1, y_{iJ} = Y,$ $x_{i+1,j} - x_{ij} \in \{0, 1, 2\}, x_{i,j+1} - x_{ij} \in \{-1, 0, 1\},$ $y_{i,j+1} - y_{ij} \in \{0, 1, 2\}, y_{i+1,j} - y_{ij} \in \{-1, 0, 1\}$
P2DHMM (1)	$x_{1j} = 1, x_{Ij} = X, y_{i1} = 1, y_{iJ} = Y,$ $\exists \{\hat{x}_1, \dots, \hat{x}_I\} : \hat{x}_{i+1} - \hat{x}_i \in \{0, 1, 2\},$ $x_{ij} - \hat{x}_i = 0, y_{i,j+1} - y_{ij} \in \{0, 1, 2\}$
P2DHMDM (1-)	$x_{1j} = 1, x_{Ij} = X, y_{i1} = 1, y_{iJ} = Y,$ $\exists \{\hat{x}_1, \dots, \hat{x}_I\} : \hat{x}_{i+1} - \hat{x}_i \in \{0, 1, 2\},$ $x_{ij} - \hat{x}_i \in \{-1, 0, 1\}, y_{i,j+1} - y_{ij} \in \{0, 1, 2\}$
IDM (0)	$x_{ij} \in \{1, \dots, X\} \cap \{i' - w, \dots, i' + w\}, i' = \left[ i \frac{X}{I} \right],$ $y_{ij} \in \{1, \dots, Y\} \cap \{j' - w, \dots, j' + w\}, j' = \left[ j \frac{Y}{J} \right],$ <b>with warp range <math>w</math>, e.g. <math>w = 3</math></b>

IDM: image distortion model; P2DHM(D)M: pseudo 2D hidden Markov (distortion) model; 2DW: 2D warping (NP-complete)

hierarchical search in nearest neighbor:

first use Euclidean distance, then more costly distance on set of closest references